LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **STATISTICS**

SECOND SEMESTER – **APRIL 2013**

ST 2815/2812 - TESTING STATISTICAL HYPOTHESIS

Date : 29/04/2013 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

Part A: Answer all the questions. Each carries 2 marks. (10 x 2 = 20 marks)

1. Distinguish between simple and composite hypotheses.
2. What is the difference between the level and size of the test?
3. What are the applications of generalized Neyman–Pearson theorem?
4. State Monotone Likelihood ratio property.
5. What is uniformly most accurate confidence bound?
6. Give an example for a three parameter exponential family of distribution.
7. Explain briefly an unbiased test and describe its applications.
8. What are nuisance parameters and how do you remove them?
9. Explain briefly the principles of likelihood ratio tests?
10. Give an example of a group with location changes.

Part B: Answer any five questions. Each carries 8 marks. (5 x 8 = 40 marks)

11. Let X_1, X_2 be iid from $B(1,\theta)$ where $\theta = \{0.1, 0.2, 0.3, 0.4\}$. Consider the problem of testing H: $\theta = 0.1, 0.3$ against K: $\theta = 0.2, 0.4$. Suppose the test function is

    $\Phi(x_1, x_2) = \{0.1$ if $(x_1, x_2) = (0, 0)$; 0.2 if $(x_1, x_2) = (0, 1)$ or $(1,0)$; 1 if $(x_1, x_2) = (1,1)$.

    Find the power function and hence compute the size of the test.

12. Let $X_1, X_2, \ldots, X_n$ be a random sample from the pdf

    $$f_\theta(x) = \theta / x^2 \text{ if } 0 < \theta \leq x < \infty.$$

    Find the most powerful test of level α for testing H: $\theta=\theta_0$ against K: $\theta=\theta_1$ ($\neq\theta_0$).

13. Suppose a random sample of size n is drawn from

    $$f(x, \theta) = \exp\{-(x-\theta)\};\ \theta < x < \infty.$$

    Does $\{f(x,\theta)\}$ belong to the exponential family? Does $\{f(x,\theta)\}$ have MLR? (4+4 marks)

14. Show that a necessary and sufficient condition for the family of distributions to have MLR property is that

$\partial^2 \log f(x, \theta) / \partial\theta\, \partial x$ exists and is non-negative.

15. Why do we require bounded completeness to prove similar tests to have Neyman structure?

16. a) Prove or disprove:

UMP test ↔ MP test.

Justify your answer. (4 marks)

b) Show that a test function is invariant if and only if it is constant on each orbit. (4 marks)

17. How do you test the equality of two Binomial Populations? Suggest some real life testing problems? (6+2 marks)

18. If for given α, $0 \leq \alpha \leq 1$, show that likelihood ratio tests and non-randomized Neyman-Pearson tests of a simple hypothesis against a simple alternative exist and are equivalent.

Part C: Answer any two questions. Each carries 20 marks. (2 x 20 = 40 marks)

19. State and Prove the existence and necessary conditions of Neyman-Pearson fundamental lemma. (12+8 marks)

20. a) Let $X_1, X_2 \ldots, X_6$ be a random sample of size 6 from $U(0, \theta)$. Derive UMPT of level 0.01 for testing H: $\theta \leq 15$ against K: $\theta > 15$ and find the cut-off point. (10 marks)

b) Construct a 95% UMACB for testing the population variance when a random sample of size n is drawn from a normal distribution with known mean. (10 marks)

21. Construct UMPT of level α for testing H: $\theta \leq \theta_1$ (or) $\theta \geq \theta_2$ against K: $\theta_1 < \theta < \theta_2$ when the random sample of size n is drawn from one parameter exponential family.

22. a) Explain in detail the concept of equivalence relation and equivalence classes. (5 marks)

b) To test H: $X \approx N(\theta, 1)$, against K: $X \approx C(1, \theta)$, a sample of size 2 is available on X. Find the UMP invariant test of H against K. (15 marks)
